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Coherence and Polarization of Polarization Speckle Generated by Depolarizers and Their Changes through Complex ABCD Matrix

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ABSTRACT

Recent research work on speckle patterns indicates a variation of the polarization state during propagation and its non-uniformly spatial distribution. The preliminary step for the investigation of this polarization speckle is the generation of the corresponding field. In this paper, a kind of special depolarizer: the random roughness birefringent screen (RRBS) is introduced to meet this requirement. The statistical properties of the field generated by the depolarizer is investigated and illustrated in terms of the 2x2 beam coherence and polarization matrix (BCPM) with the corresponding degree of coherence (DoC) η and degree of polarization (DoP) P . The changes of the coherence and polarization when the speckle field propagates through any optical system are analysed within the framework of the complex ABCD-matrix theory.

Keywords: statistical optics, laser speckle, coherence, polarization, depolarizer, ABCD matrix

1. INTRODUCTION

The speckle pattern is one of the most important concepts in optics which describes the phenomena of fine-scale, high-contrast granular light intensity distribution caused by the interference of a coherent optical field within the scattering spot¹. In the majority of studies on speckle phenomena, the random field is usually treated as a scalar random optical field. The main interest was in the statistical properties of their intensity and its changes during propagation, and it is implicitly asserted that polarization state will be maintained. Recent research work, both theoretical and experimental, indicates the variation of the polarization state during propagation and its non-uniform spatial distribution for the cross section of a speckle²⁻⁶. This special kind of speckle is named *polarization speckle*⁶ or *electromagnetic beam*⁴. To illustrate the polarization variation together with the coherence property of fields, a 2x2 beam coherence and polarization matrix (BCPM) in the spatial-temporal domain was developed by F. Gori⁷ and then expanded by E. Wolf to establish a unified theory of coherence and polarization within this framework⁸.

The basic premise of the investigation for the statistical properties of polarization speckle is the special kind of device/system which does not only diffuse the incident field but also scrambles its polarization like a depolarizer. Liquid-crystal (LC) spatial light modulators (SLMs) modulating only one of the two orthogonal field components were introduced as one of the most promising anisotropic depolarizer to give rise to a prescribed speckle field with particular spatial polarization properties⁹. The modulated field associated with the propagation in free space were investigated statistically, in terms of the BCPM implicating the polarization and coherence properties⁴. The general results for the propagation in arbitrary optical systems with finite-sized Gaussian apertures were also later derived⁵ within the framework of the ABCD matrix theorem¹⁰. However, this device will impact one of the two orthogonal field elements and thus random modulation is not introduced to the whole field. Its de-coherence ability is restricted to generating a partially coherent field. Another typical method is to implement the de-coherence and depolarization functionalities by a train of different optical devices. A polarized beam splitter is used to split the beam into two orthogonal polarized components, and then by moving phase diffusers e.g. ground glass plates, incoherent random phase retardation are introduced to one¹¹, or both split fields⁶. Subsequently, these two beams are combined again to generate the polarization speckle. This is entirely feasible. However, it is understandable that this system might be too complex and too bulky for some applications. The other problem is the uncertainty associated with the system by these multi-individual devices.

The RRBS introduced in this paper, is a thin transmitting screen made of birefringent materials with random surface roughness. This thickness fluctuation will introduce random phase retardation in the incident field over the surface plane of RRBS and diffuse it like a traditional rough screen. At the same time, the birefringence of the material will cause the random phase difference proportional to local thickness between two orthogonal wave components and thus implement the random change of the polarization state for the field over the surface plane like a depolarizer. The comprehensive and rigorous analysis of this modulation behaviour is achieved in terms of the BCPM in the vector context. The relation between the modulation behaviour and the surface structure is revealed. Based on these BCPM expressions for the modulated field after depolarizers, the polarization speckle propagating through general optical systems including inherent Gaussian apertures are analysed with the Complex ABCD matrix, and the analytical results of the propagated BCPM are provided. Related statistical properties including the coherence and the polarization including their combined changes are also discussed. By these conclusions, it is found that the RRBS proposed here provides a flexible and easy method to implement random diffusion and depolarization to the incident field and the polarization speckle field with spatially varying polarization states is successfully generated. To provide the physical insight, numerical examples are cited and plotted in figures for this model. It is expected that the analytical result about the relation between the structural features of depolarizer and the statistical properties of the propagated polarization speckle patterns will not only facilitate the applications like retrieval method of birefringent depolarizer surface structure in speckle non-contact metrology, but also promote the design and optimization of modulation devices to generate the polarization speckle with prescribed statistical properties.

2. FULLY DEVELOPED POLARIZATION SPECKLE GENERATED BY RRBS

The RRBS applied here is not a novel concept tracing back to a patent published in the US in 1969¹². However, it is the first time to be considered for the special purpose to generate the polarization speckle field. Hence, its modulation is not yet analysed theoretically. In this paper, the research work will be carried out from the introduction on its structure and the related effect on the field modulation. As named, RRBS (figure 1.(a)) is a thin transmitting screen made of birefringent materials with random surface roughness. It is placed in an optical system as illustrated in figure 1.(b).

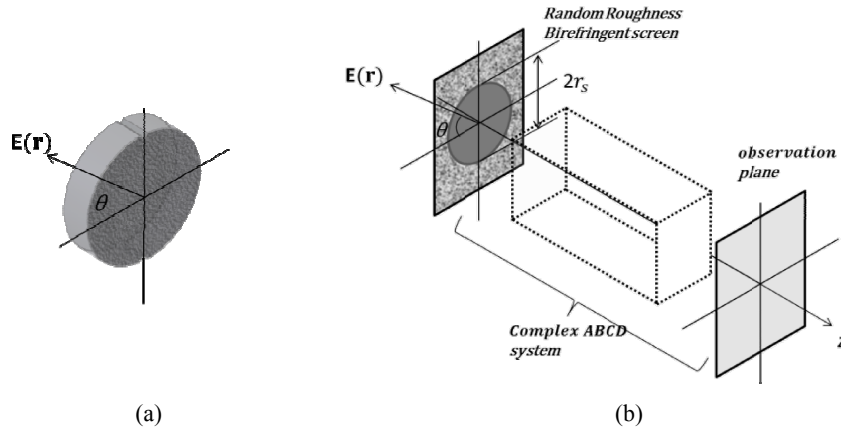


Figure 1. (a) RRBS and (b) the sketch of optical system including RRBS and complex ABCD propagation system

For a laser beam perpendicularly incident to the screen surface at point \mathbf{r} , the field vector $\mathbf{E}(\mathbf{r}) = \{E_x(\mathbf{r}), E_y(\mathbf{r})\}$ is described with the components $E_x(\mathbf{r})$ and $E_y(\mathbf{r})$ polarized vertically and horizontally. Its 2×2 BCPM is defined as⁷

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2) = \langle \mathbf{E}^\dagger(\mathbf{r}_1) \mathbf{E}(\mathbf{r}_2) \rangle = \begin{pmatrix} \langle E_x^*(\mathbf{r}_1) E_x(\mathbf{r}_2) \rangle & \langle E_x^*(\mathbf{r}_1) E_y(\mathbf{r}_2) \rangle \\ \langle E_y^*(\mathbf{r}_1) E_x(\mathbf{r}_2) \rangle & \langle E_y^*(\mathbf{r}_1) E_y(\mathbf{r}_2) \rangle \end{pmatrix}, \quad (1)$$

where the dagger \dagger depicts the Hermitian conjugate, asterisk $*$ means complex conjugate, and angular bracket $\langle \dots \rangle$ denotes ensemble average. On the basis of this concept, the parameters of special interest: viz. the DoC η and DoP P is determined by^{8,13}:

$$\eta(\mathbf{r}_1, \mathbf{r}_2) = \text{tr} \mathbf{W}(\mathbf{r}_1, \mathbf{r}_2) [\text{tr} \mathbf{W}(\mathbf{r}_1, \mathbf{r}_1) \text{tr} \mathbf{W}(\mathbf{r}_2, \mathbf{r}_2)]^{-1/2}, \quad (2)$$

$$\mathbb{P}(\mathbf{r}) = \left\{ 1 - 4 \det \mathbf{W}(\mathbf{r}, \mathbf{r}) [\text{tr} \mathbf{W}(\mathbf{r}, \mathbf{r})]^{-2} \right\}^{1/2}, \quad (3)$$

where tr and \det mean the trace and determinant of the matrix, respectively. The BCPM is also easy to be converted into Stokes' parameters¹⁴ if found necessary.

For the field passing through a RRBS well aligned with the system, the vertical and horizontal polarized wave components are modulated by different phase retardations $\varphi_m(\mathbf{r})$, ($m = x, y$), respectively. The emerging field $\mathbf{E}'(\mathbf{r}) = \mathbf{E}(\mathbf{r})\mathbf{T}(\mathbf{r})$ is equal to the product of the incident vector $\mathbf{E}(\mathbf{r})$ with the local ray-transfer matrix (or so-called Jones matrix) $\mathbf{T}(\mathbf{r})$ ^{15,16} which is defined as

$$\mathbf{T}(\mathbf{r}) = \begin{pmatrix} e^{-j\varphi_x(\mathbf{r})} & 0 \\ 0 & e^{-j\varphi_y(\mathbf{r})} \end{pmatrix} = \begin{pmatrix} e^{-jkn_x d(\mathbf{r})} & 0 \\ 0 & e^{-jkn_y d(\mathbf{r})} \end{pmatrix}, \quad (4)$$

where $d(\mathbf{r})$ is the random local thickness, k is the wavenumber, j is the imaginary unite, while n_x and $n_y = \varepsilon n_x$, ($\varepsilon > 1$) denote the corresponding refractive indices. Therefore, a random phase difference $kn_x(\varepsilon - 1)d(\mathbf{r})$ proportional to the random local thickness is introduced between two orthogonal wave components of $\mathbf{E}'(\mathbf{r})$ and the random depolarization is introduced by this depolarizer. Now, the $\mathbf{W}'(\mathbf{r}_1, \mathbf{r}_2)$ is given by,

$$\mathbf{W}'(\mathbf{r}_1, \mathbf{r}_2) = \begin{pmatrix} W_{xx}(\mathbf{r}_1, \mathbf{r}_2) \langle \exp\{j\Delta\varphi_{xx}\} \rangle & W_{xy}(\mathbf{r}_1, \mathbf{r}_2) \langle \exp\{j\Delta\varphi_{xy}\} \rangle \\ W_{yx}(\mathbf{r}_1, \mathbf{r}_2) \langle \exp\{j\Delta\varphi_{yx}\} \rangle & W_{yy}(\mathbf{r}_1, \mathbf{r}_2) \langle \exp\{j\Delta\varphi_{yy}\} \rangle \end{pmatrix}, \quad (5)$$

where W_{mn} , ($m, n = x, y$) is the matrix element of the \mathbf{W} and $\Delta\varphi_{mn} = \varphi_m(\mathbf{r}_1) - \varphi_n(\mathbf{r}_2)$ is the phase differences totally depending on the structural features of the RRBS. Here, we assume the surface roughness is in accordance with the widely accepted Gaussian-Gaussian model in order to achieve analytical expressions^{17,18}. Thus its thickness fluctuation is statistically defined in Gaussian terms with a zero mean and covariance σ_d^2 . Furthermore, with the hypothesis of wide-sense stationarity, the lateral correlation has a Gaussian dependency on the location distance $\Delta\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ with a scale r_d , i.e.

$$\langle d(\mathbf{r}_1)d(\mathbf{r}_2) \rangle = \sigma_d^2 \exp\left\{-|\Delta\mathbf{r}|^2 / r_d^2\right\}. \quad (6)$$

Since the $\Delta\varphi_{mn}$ is assumed a Gaussian process too, those ensemble average terms $\langle \Delta\varphi_{mn} \rangle$ should be the first-order characteristic function^{18,19} of $\Delta\varphi_{mn}$ given by:

$$\langle \Delta\varphi_{mn} \rangle = \exp\left\{\frac{k^2\sigma_d^2}{2}\left((n_m - n_n)^2 - \frac{2n_m n_n |\Delta\mathbf{r}|^2}{r_d^2}\right)\right\}. \quad (7)$$

The double exponential function in the above equation will not render any analytical expressions for the propagated field. Depending on the different surface roughness, this expression could be approximated adequately. For a fully developed speckle generated by a heavy surface rough RRBS with large phase covariance $s^2 = k^2\sigma_d^2 n_x^2$, and small r_d , the measure of the lateral phase correlation length $r_c^2 = r_d^2 / s^2$ will be small. Therefore, the first two terms of the inner exponential functions' Taylor expansion are retained^{20,21} as its approximation. If the linearly polarized Gaussian incident beam has a polarization angle θ and $1/e^2$ amplitude radius r_s , its BCPM $\mathbf{W}'(\mathbf{r}_1, \mathbf{r}_2)$ is approximated in the form of:

$$\mathbf{W}'(\mathbf{r}_1, \mathbf{r}_2) \cong I_o \exp\left\{-\frac{|\mathbf{r}_1|^2 + |\mathbf{r}_2|^2}{r_s^2}\right\} \begin{pmatrix} \cos^2 \theta \exp\left\{-|\Delta\mathbf{r}|^2 / r_c^2\right\} & \cos \theta \sin \theta \exp\left\{-s^2(1 - \varepsilon)^2 / 2 - \varepsilon |\Delta\mathbf{r}|^2 / r_c^2\right\} \\ \cos \theta \sin \theta \exp\left\{-s^2(1 - \varepsilon)^2 / 2 - \varepsilon |\Delta\mathbf{r}|^2 / r_c^2\right\} & \sin^2 \theta \exp\left\{-\varepsilon^2 |\Delta\mathbf{r}|^2 / r_c^2\right\} \end{pmatrix}, \quad (8)$$

The corresponding DoP and DoC are given by

$$\begin{aligned} P'(\mathbf{r}) &= \left\{ 1 - \sin^2 2\theta \left[1 - \exp\left\{-s^2(1-\varepsilon)^2\right\} \right] \right\}^{1/2}; \\ \eta'(\mathbf{r}_1, \mathbf{r}_2) &= \cos^2 \theta \exp\left\{-|\Delta \mathbf{r}|^2 / r_c^2\right\} + \sin^2 \theta \exp\left\{-\varepsilon^2 |\Delta \mathbf{r}|^2 / r_c^2\right\}. \end{aligned} \quad (9)$$

In this way, the relation between the structural features of RRBS and its simultaneous depolarization and de-coherence ability is revealed. The statistical properties of the propagated polarization speckle are sensitive to both the fine surface structure of this depolarizer introduced here and the following propagation system which will be discussed in the next section.

3. PROPAGATION OF POLARIZATION SPECKLE GENERATED BY RRBS

Within the paraxial approximation, the field propagation within an optical system with inherent Gaussian apertures could be calculated within the framework of complex ABCD theory. The output field $\mathbf{E}^o(\mathbf{p})$ in the observation plane, is the integral of the modulated field $\mathbf{E}'(\mathbf{r})$ and the Green's transform kernel $G(\mathbf{r}, \mathbf{p})$, which is given by¹⁰

$$G(\mathbf{r}, \mathbf{p}) = -\frac{jk}{2\pi B} \exp\left\{-\frac{jk}{2B}(A|\mathbf{r}|^2 - 2\mathbf{r} \cdot \mathbf{p} + D|\mathbf{p}|^2)\right\}. \quad (10)$$

Hence, the BCPM $\mathbf{W}^o(\mathbf{p}_1, \mathbf{p}_2)$ is⁵

$$\mathbf{W}^o(\mathbf{p}_1, \mathbf{p}_2) = \iint \mathbf{W}'(\mathbf{r}_1, \mathbf{r}_2) G^*(\mathbf{r}_1, \mathbf{p}_1) G(\mathbf{r}_2, \mathbf{p}_2) d\mathbf{r}_1 d\mathbf{r}_2. \quad (11)$$

In the equations above, A , B and D are the complex elements of the ABCD matrix, which is determined by the multiplication of all individual optical components' ABCD matrices. Various typical optical components' matrices were provided in reference¹⁰, and the Gaussian shaped aperture was invoked for the sake of achieving analytical integrals and therefore provides tractable results of the general transmission function¹⁰. If we omitted the Gaussian beam size r_s and properly compensated for this in the ABCD parameters, the general analytical results of the propagated field's BCPM $\mathbf{W}^o(\mathbf{p}, \mathbf{p})$ could be derived by the method introduced above:

$$\mathbf{W}^o(\mathbf{p}_1, \mathbf{p}_2) = I_o \cos \theta \sin \theta \begin{pmatrix} \cot \theta W^{ABCD}(\mathbf{p}_1, \mathbf{p}_2; r_c^2) & \exp\{-s^2(1-\varepsilon)^2/2\} W^{ABCD}(\mathbf{p}_1, \mathbf{p}_2; r_c^2/\varepsilon) \\ \exp\{-s^2(1-\varepsilon)^2/2\} W^{ABCD}(\mathbf{p}_1, \mathbf{p}_2; r_c^2/\varepsilon) & \tan \theta W^{ABCD}(\mathbf{p}_1, \mathbf{p}_2; r_c^2/\varepsilon^2) \end{pmatrix} \quad (12)$$

$W^{ABCD}(\mathbf{p}_1, \mathbf{p}_2; \mu^2)$ with parameter μ is a function defined as

$$\begin{aligned} W^{ABCD}(\mathbf{p}_1, \mathbf{p}_2; \mu^2) &= \frac{\mu^2}{4 \operatorname{Im}[A^* B]/k + \mu^2 |A|^2} \exp \left\{ \frac{-j(|\mathbf{p}_1|^2 - |\mathbf{p}_2|^2) \left\{ \operatorname{Im}[B^2] - 2 \operatorname{Im}[A^* B] \operatorname{Re}[D^* B] + \frac{k\mu^2}{2} (\operatorname{Re}[AB] - \operatorname{Re}[D^* B]|A|^2) \right\}}{(4 \operatorname{Im}[A^* B]/k + \mu^2 |A|^2) |B|^2} \right\} \\ &\exp \left\{ \frac{-(\mathbf{p}_1 - \mathbf{p}_2)^2}{4 \operatorname{Im}[A^* B]/k + \mu^2 |A|^2} \right\} \exp \left\{ \frac{-(|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2) \left\{ -2 \operatorname{Im}[B]^2 + 2 \operatorname{Im}[A^* B] \operatorname{Im}[D^* B] + \frac{k\mu^2}{2} (|A|^2 \operatorname{Im}[B^* D] - \operatorname{Im}[AB]) \right\}}{(4 \operatorname{Im}[A^* B]/k + \mu^2 |A|^2) |B|^2} \right\} \end{aligned} \quad (13)$$

Equation (12) is the primary conclusion for the propagated polarization speckle generated by the RRBS. Based on these analytical results, the corresponding DOC and DOP for different propagation systems can be calculated easily in order to study the statistical properties of the polarization speckle's coherence and polarization states.

A free space system with a propagation distance z gives an ABCD matrix having the elements $A = 1 - jz/z_R$, $B = z$, and $D = 1$, where $z_R = kr_s^2/2$ is the Rayleigh range. The $r_s = 1\text{mm}$, $r_c = 0.1\text{mm}$, wavelength $\lambda = 0.633\mu\text{m}$, and $s^2 = 100$ are taken to represent a numerical demonstration. For the DoP $P_{\text{free space}}(\mathbf{p})$ of the speckle on the observation screen, the variation with the normalized propagation distance z/z_R and the normalized location \mathbf{p}/r_s in the observation plane is

shown in figure 2. From these plots, the non-uniform distribution of the polarization on the beam cross section and its variation in propagation are demonstrated. For material with stronger birefringence, more significant DoP spatial variation will be observed. At the same time, the DoC $\eta_{free\ space}(\mathbf{p}, -\mathbf{p})$ for two symmetrical points \mathbf{p} and $-\mathbf{p}$ with respect to the normalized propagation distance z/z_R and normalized \mathbf{p}/r_s is illustrated in figure 3. The DoC increases during propagation to the far field and approaches unity, which is in accordance with the Van Cittert-Zernike theorem. The sensitivity of the DoC and the DoP to the polarization angle of the incident field is also indicated by these figures.

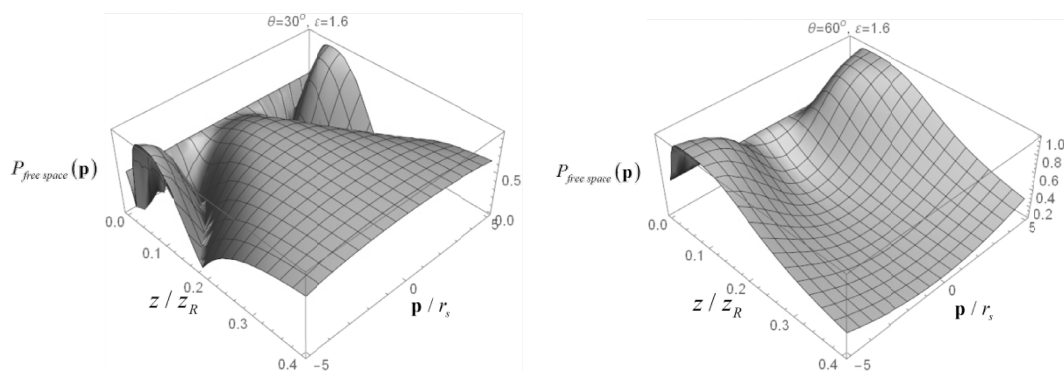


Figure 2. $P_{free\ space}(\mathbf{p})$ of the fully developed polarization speckle for a free space system for selected polarization angles

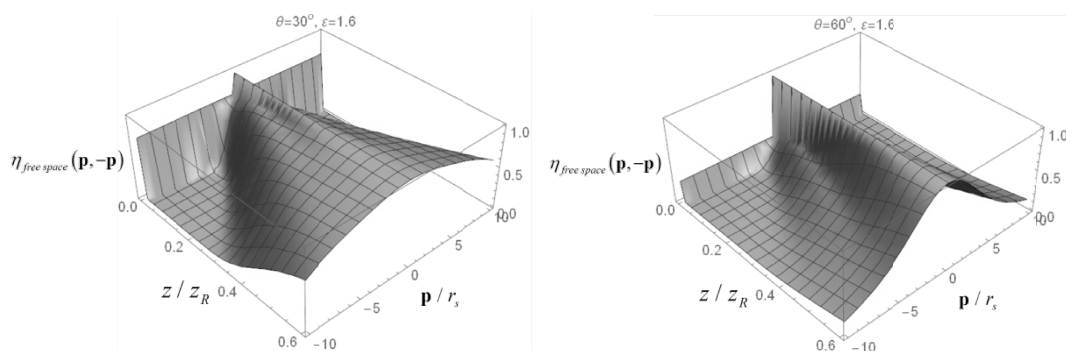


Figure 3. $\eta_{free\ space}(\mathbf{p}, -\mathbf{p})$ of the fully developed polarization speckle for a free space system for the selected polarization angles

4. DISCUSSION AND CONCLUSION

In this paper, we have proposed a generating method of polarization speckle with spatial variation of the polarization state. The ability of RRBS to simultaneously diffuse and depolarize as a compact single-device system is approved by the analytical results of corresponding BCPM of the generated field with the statistical parameters DoC and DoP. The changes of the coherence and polarization state during propagation are studied within the framework of ABCD theory for general optical systems, and specifically a free space system is cited as example to provide physical insight. Though we only discussed the fully developed polarization speckle generated by a RRBS with large phase covariance and small lateral correlation length here, other kinds of polarization speckles like the partially developed ones generated by gentle roughness models of RRBS could also be analysed in similar steps and the conclusions will be presented in our future papers. This rigorous analysis for the RRBS might help researchers in the development of the polarization speckle generating method and facilitate the improvement of speckle non-contact metrology for birefringent rough objects.

REFERENCES

- [1] Goodman, J. W., [Speckle phenomena in optics: theory and applications], Roberts & Company Publishers, (2007).
- [2] James, D., "Change of polarization of light beams on propagation in free space," J. Opt. Soc. Am. A 11(5), 1641 (1994).
- [3] Agrawal, G. P., Wolf, E., "Propagation-induced polarization changes in partially coherent optical beams," J. Opt. Soc. Am. A 17(11), 2019 (2000).
- [4] Shirai, T., Wolf, E., "Coherence and polarization of electromagnetic beams modulated by random phase screens and their changes on propagation in free space," J. Opt. Soc. Am. A 21(10), 1907–1916 (2004).
- [5] Hanson, S. G., Wang, W., Jakobsen, M. L., Takeda, M., "Coherence and polarization of electromagnetic beams modulated by random phase screens and their changes through complex ABCD optical systems," J. Opt. Soc. Am. A 25(9), 2338–2346 (2008).
- [6] Takeda, M., Wang, W., Hanson, S. G., "Polarization speckles and generalized Stokes vector wave: a review," Proc. SPIE 7387, V – 1–V – 7 (2010).
- [7] Gori, F., Santarsiero, M., Vicalvi, S., Borghi, R., Guattari, G., "Beam coherence-polarization matrix," Pure Appl. Opt. J. Eur. Opt. Soc. Part A 7, 941 (1998).
- [8] Wolf, E., "Unified theory of coherence and polarization of random electromagnetic beams," Phys. Lett. A 312, 263–267 (2003).
- [9] Scudieri, F., Bertolotti, M., Bartolino, R., "Light scattered by a liquid crystal: a new quasi-thermal source," Appl. Opt. 13, 181–185 (1974).
- [10] Yura, H., Hanson, S. G., "Optical beam wave propagation through complex optical systems," J. Opt. Soc. Am. A 4(10), 1931–1948 (1987).
- [11] Kumar Singh, R., Naik, D. N., Itou, H., Miyamoto, Y., Takeda, M., Singh, R. K., Naik, D. N., Itou, H., Miyamoto, Y., "Characterization of spatial polarization fluctuations in scattered field," J. Opt. 105010(10), (2014).
- [12] Peters, C. J., "Light depolarizer," US3433553 A, USA (1969).
- [13] [13] Wolf, E., [Introduction to the Theory of Coherence and Polarization of Light], Cambridge University Press, (2007).
- [14] Wolf, E., "Polarization invariance in beam propagation," Opt. Lett. 32(23), 3400 (2007).
- [15] Mandel, L., Wolf, E., [Optical Coherence and Quantum Optics], Cambridge University Press, (1995).
- [16] Goodman, J. W., [Introduction to Fourier Optics], McGraw-Hill 35 (1988).
- [17] Dainty, J. C., Ennos, A. E., Franqo, M., Goodman, J. W., McKechnie, T. S., Parry, G., [Laser Speckle and related phenomena], Springer-Verlag Berlin and New York (1975).
- [18] Goodman, J. W., [statistical optics], Wiley-Interscience, New York (2000).
- [19] Wasserman, L., [All of Statistics: A Concise Course in Statistical Inference], Springer Science & Business Media (2004).
- [20] Hyde, M. W., Basu, S., Spencer, M. F., Cusumano, S. J., Fiorino, S. T., "Physical optics solution for the scattering of a partially-coherent wave from a statistically rough material surface", Opt. Express 21(6), 6807–6825 (2013).
- [21] Rose, B., Imam, H., Hanson, S. G., Yura, H. T., "Effects of target structure on the performance of laser time-of-flight velocimeter systems," Appl. Opt. 36(2), 518–533 (1997).